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Discussion

# Response to the paper by Y.Z. Chen "Singular integral equation method for the solution of multiple curved crack problems" (International Journal of Solids and Structures 41 (2004) 3505–3519)

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Multiple curvilinear cracks being common in solids, the paper is of value for computational fracture mechanics. It contributes to this field by the popularization of an efficient numerical tool suggested by using the complex variables. The popularization is useful in view of still insufficient appreciation of the computational advantages of the complex variable equations in plane elasticity problems for media with structure. Meanwhile, the paper has flaws, which I feel necessary to discuss.

*On the claimed 'new method'.* The author claims (p. 3507) that in his paper "a new method for cracks of general curvature that is not restricted to slightly curved cracks is developed". He explains the essence of the method as follows (p. 3507): "The crack length is taken as the coordinate along the crack and the crack configuration is mapped on a real axis in an interval (−a, a)." He continues: "The proposed method is called the 'curve length method' hereafter."

Actually, the method is but the well-established method of mechanical quadratures (MMQ) systematically and successfully used by M.P. Savruk and his co-workers for more than 20 years (see, e.g., the monographs (Savruk, 1981; Savruk et al., 1989; Linkov, 1999, 2002), containing many other references). Using the length of a crack as a parameter for mapping is just one of infinite options suggested by this method; this option among others was used by Savruk. Although the paper contains references to the monograph (Savruk, 1981), still the method is called 'new'.

*Motivation.* When discussing in Introduction singular equations, Y.Z. Chen writes (p. 3506): "In general, the range of application of previously obtained solutions for the curved crack problem is however not satisfactory." This statement does not look correct in view of the results by Datsyshin, Ioakimidis, Savruk (for singular equations, 1976–2003), by Dobroskok, Koshelev, Linkov and Mogilevskaya (for hypersingular

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equations, 1990–2004), who developed computer codes of general use for arbitrary curvilinear cracks. Many of the numerical results are reviewed in the monographs (Savruk, 1981; Savruk et al., 1989; Linkov, 2002). Unfortunately, the paper does not contain discussion of these results; instead, the author writes about obvious limitations of the perturbation method.

Furthermore, the author writes (p. 3506): ‘‘Generally, the integration rule for the hypersingular integral along a curve is rather complicated.’’ The latter statement contradicts the conclusion specially emphasized in the book (Linkov, 1999, 2002): evaluation of the complex variable hypersingular integral along an arbitrary curvilinear contour is as easy as evaluation of the singular integral. Although the Russian edition of the book is referred to by Y.Z. Chen, there are no arguments in the paper to support his statement cited above.

*Review.* It would be natural when writing on the well-established singular equations to refer to either the papers where the equations have been firstly derived or to monographs containing a detailed historical review. Instead, the author refers to his paper of 1991 for the singular equations in terms of dislocations and only to his paper of 1999 for the singular equations in terms of the resultant force (p. 3506). Note that the *both* types of equations for multiple curvilinear cracks have been derived and studied yet in 1974 (Linkov, 1975; Eqs. (8) and (5)). On the other hand, a detailed historical review of existing equations, including the discussed, and their interrelation are given in Linkov (1999, 2002).

A similar flaw appears also when the author writes on hypersingular equations (p. 3506). In fact, he attributes their derivation to himself (1993). Meanwhile, they have been derived yet in 1990 (Linkov, 1990; Linkov and Mogilevskaya, 1990). These references are given in Linkov (1999, 2002), as well.

There is no other discussion of predecessors in the review given in the paper.

*Theoretical contents.* A significant part of the paper (p. 3507–3513) contains well-known derivation of Eqs. (10) and (14) and well-known formulae of the MMQ (see, e.g. Savruk, 1981; Savruk et al., 1989; Linkov, 1999, 2002). There was no need in reproducing these results: it would be sufficient to make a reference and to note that the author uses a particular choice of mapping.

*Incorrect statements.* Besides the mentioned incorrect attribution of theoretical results, the paper contains a misleading statement. In the introduction and also in the conclusion to his paper (p. 3518) the author writes that previously ‘‘no integration rule was suggested to evaluate a singular integral along a curve’’. This statement is not correct. As is easily seen and has been emphasized in my book (Linkov, 1999, p. 298; Linkov, 2002, p. 199): ‘‘In complex variables … evaluation of singular and hypersingular integrals over *curvilinear* elements is as simple as their evaluation over *straight* segment of the real axis.’’ Recurrence formulae for such evaluation are simple; in particular, they are given in the cited book.

*Examples.* Any of existing codes for multiple curvilinear cracks, developed, for instance, by Datsyshin, Dobroskok, Koshelev, Mogilevskaya, Savruk and other authors, may easily produce numerous examples for various crack number, configurations and mutual positions. Hence, examples, when presenting them, must correspond to the purpose of a particular paper.

The examples, given in the paper, do not meet this requirement. They concern with couples of *circular-arc* cracks. But all the integrals over a circular-arc element are evaluated analytically for standard approximations of the density (Mogilevskaya, 1996). Thus, the mapping to the real axis used by Y.Z. Chen in the frames of the MMQ misses this advantage. However, the MMQ gains in using Gaussian quadratures. The examples would be justified if the numerical results obtained by the MMQ were compared with those provided by the mentioned analytical evaluation of integrals. Without such comparison, the presented examples are not sufficiently informative.

In conclusion, it seems appropriate to note that the *singular* integral equations, popularized in the paper, are not the only choice for efficient solving the considered problem. As explained in (Linkov, 1999, 2002), the *hypersingular* equations suggest even better choice, especially when cracks are closed and we need to account for irreversible deformations on surfaces.

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